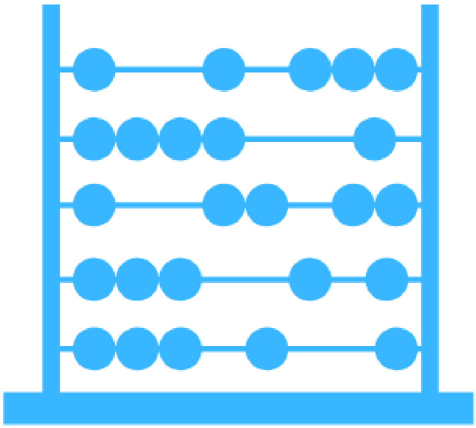
A PARLIAMENT STREET RESEARCH PAPER

THE MATHS REVOLUTION

The case for traditional arithmetic



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The Maths Revolution:
The case for traditional arithmetic
By Tom Burkard and Colin McKenzie

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Tom Burkard is a Visiting Professor of Education Policy at the University of Derby and is one of Britain's leading authorities on synthetic phonics. After teaching remedial literacy skills at Costessey High School in Norwich, he wrote the Sound Foundations decoding and spelling programmes, which proved highly effective in independent evaluations by local authority officials in Gloucestershire and Southampton. Sound Foundations books are now used in well over 1,000 primary and secondary schools in the UK. He is the author of numerous articles and policy documents on education and was awarded a DPhil by Published Works by the University of Buckingham in 2016. In April 2018 he was asked to join Ofsted's Maths Advisory Group, where he worked on the new Inspection Guidance for primary and secondary Maths. He is a member of the NASUWT.

Colin McKenzie has achieved excellent results in Physics to GCSE and Chemistry to A Level for the last 16 years. He has trained teachers in basic literacy skills and produced resourced packages for scientific literacy and numeracy. His science resources were shared throughout Cumbria, where they were credited by LEA officials as having played a major role in raising Science results across the county. As well as being Head of Chemistry at Netherhall School, he played a leading role in improving results at West Lakes Academy with a coordinated programme of testing and intervention. More recently he was Head of Science at Pendle Vale College in Lancashire, where his research included a morphemic approach to scientific literacy and the research into automatic recall of number bonds that led to the Fast Maths intervention programme. He is now Director of McKenzieTutor in Lancashire, where he works with parents, schools and community organisations to design and deliver innovative courses of instruction which are unconstrained by the bureaucracy that absorbs so much of teachers' time and energy.

Introduction

Maths education in England is now at a critical juncture. For the last generation, the Maths curriculum has reflected a belief that calculators and computers have rendered traditional arithmetic obsolete, and that teaching should prioritise mathematical reasoning and problem-solving. Pupils are now expected to 'think like a mathematician' even if they can't solve simple arithmetic problems.

However, research from the cognitive sciences has now established beyond doubt that higher-order thinking skills cannot be taught to pupils who lack a very firm base of the relevant knowledge. Ofsted's *Bold Beginnings* [1]report reflects this understanding--even the NCETM (National Centre for Excellence in the Teaching of Mathematics) accepts that pupils should achieve automaticity in basic curricular objectives before progressing to more complex procedures and concepts.

1 https://www.gov.uk/government/publications/reception-curriculum-in-good-and-outstanding-primary-schools-bold-beginnings

Although we have previously expressed strong reservations about Ofsted's role in assessing the quality of teaching and learning, [2] we have long recognised that maths teaching is now hopelessly inadequate--and that disadvantaged children are the main victims of misguided pedagogy. Tom Burkard agreed to join Ofsted's Maths Advisory Group because it was the only significant force in England's educational establishment that unequivocally recognised the need for change.

At the same time, Colin McKenzie was Head of Science at Pendle Vale College near Burnley, a school with a strong research culture. Recognising the need for accurate formative assessment to identify gaps in pupils' basic arithmetic skills, we worked with Head Steve Wilson and Head of Maths Alison Watson to devise and administer two simple tests--a timed test of basic multiplication facts up to 12 x 12 and an arithmetic test. This consisted of 12 questions similar to those on the KS2 Arithmetic Paper, which is administered to all Year 6 pupils in England. We tested a total of 565 pupils in Year 7 and Year 8.

The results confirmed our worst suspicions: of the 212 pupils in the Year 7 intake for the current year (2018-2019), only 16 pupils were fully fluent in their multiplication tables, and 35 pupils had scores of 2 or less on the arithmetic test. Ten were unable to answer a single question.

At the same time, the results didn't surprise us at all. The fact that a few pupils did have very good scores is a credit to the primary schools that feed Pendle Vale, all of which were rated 'Good' in their most recent Ofsted inspection. Rather, the problem is with the National Curriculum. As a primary maths specialist told us,

> The curriculum moves so fast, you just don't have time for things to stick. Then you end up cramming them like mad for their SATs at the end. It's not surprising if most pupils don't remember any of it when they never even learned it properly in the first place.

It's not just that the National Curriculum for Primary Maths has too many objectives. Rather, it reflects the priority of 'concepts' over procedures. Although there are some scholarly papers to support this practice, more recent research rejects it. Hung-Hsi Wu, a professor of Mathematics at the University of California, argues that

> The desire to achieve understanding in a technical subject such as mathematics while minimizing the component of skills is a most human one. There are situations where efforts to this effect are called for and, indeed, brilliantly executed. ...[yet] In the context of school mathematics, however, such a desire cannot be indulged without doing great harm to students' education. 3

Perhaps the most valuable insights we gleaned from our tests at Pendle Vale was found in analysing pupils' workings. Of those who used the traditional algorithm for multiplication, 61% answered our first multiplication question correctly, as compared to 25.5% of those who used other methods. The universal use of 'partitioning' strategies to solve percentage problems, which presumably have been taught to impart 'number sense', produced exceptionally high error rates, even with pupils in the top sets.

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2 http://parliamentstreet.org/wp-content/uploads/2017/02/Free-Schools-For-A-Free-Society.pdf

3 Wu, H. (1999). Basic skills versus conceptual understanding. *American Educator*, *23*(3), 14-19

Despite recent attempts by ministers to introduce practices associated with traditional arithmetic, the structure of the National Curriculum for primary Maths is overwhelmingly dominated by progressive thinking. The problem is less with the overall content than the progression; concepts--which are more easily understood after pupils have learned the relevant procedures--overload the curriculum with needless complexity. Traditional algorithms are relegated to a non-statutory appendix and too much time is devoted to teaching a range of primitive calculating strategies that supposedly teach pupils multiple strategies for 'problem solving'.

In theory, this shouldn't be a problem: schools are now required to write their own curricular progression for all subjects. However, the safe course is to follow the sequence laid down in the National Curriculum as closely as possible. Even Academies and Free Schools, which are not legally bound by the NC, are likely to assume that this is what Ofsted expects. In any event, the advice published by the NCETM and disseminated through our 'Maths Hubs' is tightly tied to the sequence laid out in the NC.

We have strongly urged that schools should provide objective evidence that pupils have achieved a high degree of automaticity in each topic before moving on to more complex material. Although this practice is supported by recent research, we are completely opposed to the imposition of top-down strategies devised by 'experts'--even when we are the experts in question. As Tom Burkard argued ten years ago, top-down strategies don't work. 4 Just as the synthetic phonics revolution was started over a generation ago by a few brave teachers who rejected the progressive whole language theory that was vigorously promoted in teacher training, [5] we will never get methods that work in real, existing, classrooms unless working teachers are driving that change.

Primary schools that produce outstanding results in Arithmetic SATs--especially for the most disadvantaged pupils--should be given the means and the opportunity to share their expertise. Likewise, secondary schools should be encouraged to use Arithmetic SATs as a baseline for catch-up programmes and similar tests to evaluate pupils' progress. Above all, Ofsted should shift its emphasis from criticising failing schools--which seldom results in lasting improvements--and concentrate on analysing and publishing the strategies employed by our best schools. If ministers and officials are to have any hope of 'closing the gap' for disadvantaged pupils, Maths teaching in England will need to improve dramatically.

Summary:

1. **The Pendle Vale tests--**although the KS2 Arithmetic Test is a very good test, pupils only need score 50% to meet the standard set by ministers. Most of the pupils we tested exceeded this--even though many had little mastery of even the most basic mathematical procedures.

2. **The 'Best Practice' Fallacy**--rather than relying on a consensus of expert opinion, pedagogy should be driven by teachers and schools that produce the best results. A National Curriculum can perhaps be justified in the sense of specifying academic objectives, but insofar as it attempts to mandate 'Best Practice', it stifles initiative.

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4 https://policyexchange.org.uk/publication/every-child-a-reader/

5 https://www.cps.org.uk/press/media-coverage/q/date/2017/12/05/success-for-cps-policy-as-teaching-phonics-boosts-readi/

3. **Concepts are more easily understood once relevant procedures have been mastered--**this informs practice in the Far East, where pupils outperform those in the West on both PISA and TIMSS Maths tests. The progression in England's National Curriculum is an egregious example of trying to teach conceptual knowledge first.

4. **Topics should be thoroughly mastered before moving on to more complex material--**unfortunately, our NC uses a 'spiral curriculum', where each topic is studied in more depth each year. In practice, learning is so superficial that many pupils--if not most-- forget the previous year's content.

5. **Constructivism is based on false premises--**young children don't really 'invent' their own Maths. Rather, they are taught a range of inefficient strategies which place an unnecessary burden on their working memories. Disadvantaged pupils frequently become completely disengaged.

6. **Primitive strategies don't work**--the results of our tests clearly demonstrate that these are so complex (and tedious) that all but the brightest pupils become hopelessly bogged down. By contrast, pupils who used conventional (or 'basic') algorithms for multiplication had much better results.

7. **Number sense**--is generally considered a precondition for learning Maths. Our tests demonstrate that insofar as this mysterious quality can be defined, the National Curriculum has spectacularly failed to deliver it.

8. **Catching up**--if ministers and officials are serious about 'closing the gap', we need to develop effective catch-up programmes for pupils who have already fallen behind. A 2016 intervention at Pendle Vale suggests a radical change of pedagogy.

9. **Recommendations**--For once and for all, we have to abandon the concept that officials should rely upon a consensus of 'expert' opinion to dictate best practice--rather, we need a level playing-field so that our best teachers can demonstrate what works in real, existing classrooms.

1. The Pendle Vale tests:

Alison Watson, the Pendle Vale Maths curriculum leader, had already been securing improved results year on year with strong discipline and a highly focused, teacher-led approach to maths lessons. She confirmed that her teachers were also spending a lot of time with intervention and catch-up sessions for students who either never had the basic skills to begin with, or who had lost them as the curriculum inexorably moved ahead. However, she also recognised the need for tests for formative assessment of the gaps in pupils' learning--these would allow her department to plan interventions to make the best use of limited time and resources.

The first priority was automatic recall of number bonds, better known as learning the 'times tables'. This is a foundation skill somewhat akin to good phonological skills in reading, but for over a generation the maths education establishment dismissed this as mere 'rote learning'. Research has established that nothing could be further from the truth:

> Certain procedures and algorithms in mathematics are so basic and have such wide application that they should be practiced to the point of automaticity. Computational

> fluency in whole number arithmetic is vital. Crucial ingredients of computational fluency are efficiency and accuracy. Ultimately, fluency requires automatic recall of basic number facts: by basic number facts we mean addition and multiplication combinations of integers 0 through 10. [6]

Our recall test consisted of randomly presented two-digit problems from 1x1 to 12x12 inclusive. Pupils were instructed to complete the answers in sequence and were told not to skip any items. They were scored on the basis of how many correct answers they wrote in 60 seconds. Of the 566 pupils we tested, only 10.8% reached the accepted standard of 40 correct responses per minute. [7] A further 9.9% were judged to be partially fluent--i.e., those with scores between 31 and 39 correct answers. In all probability, these pupils were fluent in most number bonds, but not all. Scores of <10 indicate that the pupil has automatic recall of few (if any) number bonds:

Year group	No. of pupils total	No. of pupils scoring >39	No. of pupils scoring 30-39	No. of pupils scoring <10
2016	189	20	24	38
2017	165	25	20	31
2018	212	16	23	70

Nonetheless, the Year 4 times table check which will become statutory in June 2020 will require only ten correct responses per minute. [8] All but 70 of the 212 pupils in Pendle Vale's 2018 cohort were able to meet this standard. To have set a standard approaching even partial fluency would clearly have been politically impossible--ministers had enough trouble getting this far.

It should be noted that all pupils were tested between July and September 2018, so the earlier year groups will have had the advantage of either one or two years' additional instruction. In all probability, this is a major factor in the reduction in the percentage of pupils with recall scores <10.

Even scores of >39 do not necessarily indicate that pupils have unmediated recall. Some pupils practice primitive calculating strategies (such as partitioning, repeated addition or doubling) to the point where they operate very rapidly, but at the same time this may place enough additional burden on working memory to impede performance on higher-order tasks. This is suggested by Price et al:

> Using functional magnetic resonance imaging, we correlated brain responses to single digit calculation with standard scores on the Preliminary Scholastic Aptitude Test (PSAT) math subtest in high school seniors. PSAT math scores, while controlling for

6 Ball, D. L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J., Schmid, W., & Schaar, R. (2005). Reaching for common ground in K-12 mathematics education. *Notices of the AMS*, *52*(9), 1055-1058.

7 Howell, K. W., & Nolet, V. (2000). *Curriculum-based evaluation: Teaching and decision making*. Belmont, CA: Wadsworth; Hasselbring, T. S., Goin, L. I., & Bransford, J. D. (1988). Developing math automatically in learning handicapped children: The role of computerized drill and practice. *Focus on Exceptional Children*, *20*(6).

8 https://thirdspacelearning.com/blog/year-4-multiplication-times-tables-check-new-assessment-framework-2020/

> PSAT Critical Reading scores, correlated positively with calculation activation in the left supramarginal gyrus and bilateral anterior cingulate cortex, brain regions known to be engaged during arithmetic fact retrieval. At the same time, greater activation in the right intraparietal sulcus during calculation, a region established to be involved in numerical quantity processing, was related to lower PSAT math scores. [9]

This raises the possibility that learning number facts by rote may in fact be the best method. As will be discussed later, this approach can in fact be highly motivating for pupils, but it is unlikely that many primary schools would countenance a method which has long been regarded by progressive educators as a barbaric relic of our Dickensian past. Of course, rote learning is nothing more than learning something so that it may be reliably recalled at a later date. No one has ever explained why this should preclude understanding--in fact, understanding is impeded if the pupils do not have automatic recall of relevant declarative and procedural knowledge:

> Basic multiplication facts are considered to be foundational for further advancement in mathematics. They form the basis for learning multi-digit multiplication, fractions, ratios, division, and decimals (Elkins, 2002; Howell & Nolet, 2000; Kilpatrick et al., 2001; Norbury, 2002). Many tasks across all domains of mathematics and across many subject areas call upon the recall of basic multiplication facts as a lower-order component of the overall task. [10]

Our second test was the Arithmetic Test, which consisted of 12 questions similar to those on the 2017 Key Stage 2 Arithmetic Test. In fact, this is a very good test--or at least it would be if the 'pass' mark were considerably higher than 50% and partial credit weren't given for wrong answers. And of course it is inherently corrupt and corrupting when teachers have to administer high-stakes tests which are used as a measure of their school's performance. Although we suspect that outright cheating is relatively rare, it would hardly be surprising if a few schools didn't bend the rules ever so slightly. Altogether, it's a very convenient way for governments to get the results they need to prove that their policies are 'working'.

With results below, once again, we must stress that all three year groups were tested between July and September 2018, so the earlier year groups will have had the advantage of an additional year or two of instruction:

Question	2016 intake	2017 intake	2018 intake
1. 3.9 + 46	79.4%	69.1%	63.0%
2. 345 - 67	81.0	84.2	84.6
3. 438 x 63	41.3	43.0	42.8
4. 584 divided by 8	62.4	48.5	60.1
5. 36% of 14	38.1	23.0	16.8
6. 4/5 - 1/4	55.0	51.5	46.6
7. 3.6 + 2.517	82.0	81.8	80.8
8. 5002 - 834	66.7	64.2	71.2
9. 65.8 x 4.7	7.9	10.8	5.3
10. 486 divided by 27	34.9	37.6	32.2

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9 Price, G. R., Mazzocco, M. M., & Ansari, D. (2013). Why mental arithmetic counts: brain activation during single digit arithmetic predicts high school math scores. *Journal of Neuroscience*, *33*(1), 156-163.

10 Wong, M, & Evans, D (2007) Improving basic multiplication fact recall for primary school students *Mathematics Education Research Journal*, *19*(1), 89-106. https://files.eric.ed.gov/fulltext/EJ776257.pdf

11.	47% of 563	18.0	15.8	11.1
12.	1 ½ x ¾	24.3	20.6	15.4
	Average score	49.3%	47.0%	44.5%

Although we cannot be sure how representative these scores are, it should be noted that Pendle Vale's feeder schools have SATs results at or near national norms. When one allows for the fact that we did not give partial credit for wrong answers, it is very likely that the scores for the 2018 intake would be fairly close to the 50% 'pass' rate if we had done so. At the same time, we need to reflect that the above scores are averages: over 15% of these pupils were unable to answer the easiest subtraction problem.

These results suggest that ministers and officials have a rather large mountain to climb if they are to have any chance of 'closing the gap' for disadvantaged children. In her analysis of the results of the National Child Development Study, Alison Wolf found that

> Poor literacy and poor numeracy—especially the latter—have a devastating effect on people's chances of well-paid and stable employment. Moreover, this is not just because people with poor skills tend to have few GCSEs or other formal qualifications. Even after controlling for these, the effects of low skill levels are major and evident. [11]

2. The 'Best Practice' fallacy:

Ministers would do well to consider the precedent of the 2006 Rose Review, which supplanted the 1997 National Literacy Strategy. The NLS was a carefully-crafted compromise designed to appease all of the factions in the 'Reading Wars', but at the same time it reinforced the progressive orthodoxy that teachers should select the techniques that best suited each pupil's perceived 'learning style'. By 2006, evidence from the cognitive sciences had established beyond question that all good readers could translate letters to sound effortlessly and automatically, and that a substantial minority of children would not learn these skills unless they were explicitly taught.[12] At the same time, evidence from Clackmannanshire and West Dunbartonshire had demonstrated that systematic teaching of synthetic phonics could all but eliminate reading failure--as well as improving performance for even the most able pupils.[13]

Instead of calling for a review of expert opinion, ministers took the almost unprecedented step of looking at the schools in England that had the best results in early reading. In other words, they were looking at what actually worked in real, existing classrooms rather than what the experts believed to be the most effective pedagogy. Unsurprisingly, the ten schools in England with the best results used a synthetic phonics approach. The 2006 Rose Review swept away all 315 requirements in the old National Literacy Strategy, which no doubt helps to explain why such a radical change was accepted by ordinary working teachers. While conducting INSET in various local authorities, Tom Burkard was struck by the willingness of local authority advisers and classroom teachers to embrace synthetic phonics. As a teacher in Gloucestershire remarked, "We know that what we're doing now isn't working".

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11 Wolf, A., (2002) *Does Education Matter? myths about education and economic growth,* Penguin Books, London, p.34

12 Hoover, W. A., & Gough, P. B. (1990). The simple view of reading. *Reading and writing*, *2*(2), 127-160; Rose, J. (2006). Independent review of the teaching of early reading.

13 Burkard, T (March 1999) *The End of Illiteracy? The Holy Grail of Clackmannanshire,* Centre for Policy Studies; Burkard, T (Nov. 2006) *A world first for West Dunbartonshire: the elimination of reading failure,* Centre for Policy Studies

What teachers are doing now most definitely is working--especially for disadvantaged children. As Colin McKenzie observed in an earlier paper written for *Schools Week*,

> The fact that almost all children are now able to read well enough to access teaching materials has been more transformative that anything else over the past 15 years. It's a totally different world at the 'lower end' now than it was then.

Although the parallel with the Rose Review is not exact, Ofsted's *Bold Beginnings* report was likewise an analysis of how basic skills are taught during Reception Year in our best schools. Yet in relation specifically to Maths teaching, we lack an organised and coherent forum like the Reading Reform Foundation, so there are no clear alternatives to the pedagogy now disseminated by our Maths Hubs.

Yet at the same time, there can be no mistaking that a similar revolution is under way in Maths; the progressive ideology that informs the structure of the National Curriculum is now looking as dated as the 'whole language' and 'real books' pedagogy was in the first years of this century. This may well embolden our best schools to develop and enunciate alternatives, but ministers--and Ofsted--should consider actively encouraging schools to do so. In today's educational climate, sticking one's head above the parapet can be a career-threatening move.

This should be a permanent revolution--we really do need to stop thinking in terms of defining 'best practice'. We do not presume for a minute that our ideas should be privileged in any way--the sources we quote are usually (if not always) contested, and all we ask is the opportunity for teachers such as ourselves to compete on a level playing field. We should reflect that medicine has advanced considerably since bleeding patients was considered best practice for a wide range of conditions, and that even now medical researchers are continuously searching for even better treatments for all manner of physical and psychological complaints.

<u>3. Concepts are more easily understood once relevant procedures have been mastered:</u>

Compared to the National Curriculum for Maths in Years 1, 2 and 3, the old National Literacy Strategy was simplicity itself. For a start, none of the 315 requirements of the NLS were statutory: it was left to local authorities to implement them as they saw fit. By contrast, the National Curriculum for Maths has no less than 97 statutory requirements in the first three years, and a further 57 headings classed as "notes and guidance (non-statutory)". Many of these are vague and open-ended, such as "[Pupils] continue to interpret data presented in many contexts". Others are laundry lists of related learning objectives, such as these activities for Year 2:

> Pupils use fractions as ‘fractions of’ discrete and continuous quantities by solving problems using shapes, objects and quantities. They connect unit fractions to equal sharing and grouping, to numbers when they can be calculated, and to measures, finding fractions of lengths, quantities, sets of objects or shapes. They meet 3/4 as the first example of a non-unit fraction. [14]

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14 https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/335158/PRIMARY_national_curriculum_-_Mathematics_220714.pdf

All this comes before pupils have even learned the simple procedure for adding fractions with the same denominator (e.g., 1/4 + 3/4)--this won't happen until Year 3, by which time most pupils will almost certainly have forgotten the above conceptual knowledge. Even then, the NC does not introduce the addition and subtraction of fractions with different denominators--such as 1/2 + 1/3--until Year 6.

These examples illustrate two of the greatest structural weaknesses of the progression in the National Curriculum--not only are concepts introduced before pupils have the concrete procedural knowledge needed to make them readily comprehensible, but topics are spread out over six years in what is known as a 'spiral' curriculum. Instead of thoroughly mastering procedural and conceptual knowledge in each topic before moving on to more complex material, a spiral curriculum revisits each topic annually, theoretically deepening understanding each year.

However, even in subjects where topics do not form a logical progression, much (if not all) of the previous year's content will be forgotten if there is not enough time to study them to the point where they are for all intents and purposes permanently stored in long term memory. With a curriculum as overloaded as England's National Curriculum for Maths, even the most able pupils struggle to achieve automatic recall of the most basic knowledge. Less able pupils fail to acquire even that, and each year they become progressively more disengaged. The implications of the spiral curriculum will be considered in detail later.

It must be stressed that there is little direct evidence to support the teaching of skills before concepts. Rittle-Johnson et al review the literature:

> A review of the empirical evidence for mathematics learning indicates that procedural knowledge supports conceptual knowledge, as well as vice versa, and thus that the relations between the two types of knowledge are bidirectional. However, alternative orderings of instruction on concepts and procedures have rarely been compared, with limited empirical support for one ordering of instruction over another. ..Future empirical research on the effectiveness of different ways to sequence instruction on concepts and procedures is greatly needed. [15]

Nonetheless, there is considerable indirect evidence that concepts are more easily grasped after the relevant procedures have been mastered. Pupils in the Far East outscore European pupils in Maths on both international tests (PISA and TIMSS); Norton and Zhang observe that

> The Chinese approach considers that once basic knowledge and skills have been acquired, problem solving can be accomplished without students being hindered by computational limitations. Immediate understanding is not necessarily assumed. Thus, there has been an insistence on fast and accurate calculation of arithmetic operations with whole numbers and fractions, then with algebraic expressions... The product of understanding may come some time after rules have been memorised and used in different contexts...Li (2004a) describes four steps in Chinese students' approach to learning:
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> 1. initially they commit the material to memory;
> 2. next, they seek to understand the intention, style and meaning of the material;

15 Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review*, *27*(4), 587-597.

3. then they try to apply their understanding to situations that call for use of such knowledge;
4. and finally they enter a deeper level of questioning and modification of the original material. [16]

Whether this approach is the key factor in the superior performance of pupils in the Far East is of course a matter for intense debate, but Norton and Zhang conclude their advantage is not trivial:

> ...the content or substance of Far Eastern curricula is very similar to that of the West, the difference being that for East Asian students, a process of acceleration occurs where students, including Chinese students in Year 8, seem to be expected to be proficient in mathematics at a level that is not expected until Year 10 or later in the West. [17]

A Nuffield Foundation report indicates that a similar approach is common to other high-performing Asian countries:

> ...researchers took responses to 20 items from the (third) TIMSS eighth grade (13-year-olds) teacher questionnaire they considered elicited views on: the nature of mathematics; mathematical pedagogy; and learning mathematics. The items were grouped into two categories according to whether or not they indicated a perception of mathematics as procedural and algorithmic, or as conceptual and forming a connected whole (Philippou and Christou, 1999). The analysis of teacher responses on these items broadly indicated that teachers from the four Eastern Asian countries surveyed (Japan, South Korea, Hong Kong and Singapore) had responded in ways consistent with an algorithmic (procedural) view of mathematics and its teaching. Teachers from the four European countries (French Belgium, Sweden, Germany, and England) gave responses classified as towards the conceptual end of the classification. [18]

Obviously, there are other possible explanations for the superior performance of Asian pupils, not the least being the cultural emphasis on respect for elders and discipline. Nonetheless, it would seem highly unlikely that giving skills precedence over concepts disadvantages pupils in any way. As Wu explains,

> ...in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding. ..Conceptual advances are invariably built on the bedrock of technique. Without the quadratic formula, for example, the theoretical development of polynomial equations and hence of algebra as a whole would have been very different. [19]

This would suggest that teaching skills first would greatly facilitate the acquisition of conceptual knowledge, and hence reduce the amount of time devoted specifically to the latter.

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16 Norton, S., & Zhang, Q. (2013). Chinese Students' Engagement with Mathematics Learning. *International Journal for Mathematics Teaching & Learning*.
17 ibid
18 Askew, M., Hodgen, J., Hossain, S., & Bretscher, N. (2010). *Values and variables: Mathematics education in high-performing countries*. London: Nuffield Foundation.
19 Wu, op cit

It is of course totally unrealistic to expect the National Centre for Excellence in the Teaching of Mathematics (NCETM) and our Maths Hubs to reverse long-standing practices in England's schools. Nor can we honestly say that miraculous results would instantly follow a reordering of priority between concept and procedure. Such change as may occur will only happen as a result of working teachers demonstrating to their colleagues that change works in the best interests of their pupils. We believe that the cognitive revolution will inspire many schools in England and elsewhere to restore the best features of a traditional, skills-first Maths curriculum, and that the effect will be especially profound for our least-advantaged pupils. Just as research in the cognitive sciences played a key role in the synthetic phonics revolution, it is now beginning to have a major impact in the Maths revolution:

> Throughout the 1980s and 1990s, many proponents of teaching for conceptual understanding referred to developments in cognitive psychology in their arguments, declaring constructivist views on learning whereby it was argued that student activities should primarily grow out of problem situations instead of computational training (NCTM 1989). Researchers in cognitive science and psychology have declared opposite views in many cases, arguing that some of the fundamental principles in mathematics education research were, if anything, a misapplication of current views in cognitive science. To exemplify this, a position they attack is the view that mathematics should be learned in contexts and in a holistic manner, whereby it does not simply become an accumulation of concepts and skills. What cognitive science instead indicates (according to Anderson et al. 1995) is that mathematics should be decomposed into small components that can be studied and practiced in decontextualized settings; it is then the role of the educator to create sequences of tasks and activities that give the student the opportunity to create a larger whole from these components. [20]

4. Topics should be thoroughly mastered before moving on to more complex material:

Put simply, traditional maths assumes that the curriculum builds skills systematically. For instance, once pupils can identify units, tens, hundreds and thousands, they are ready to move on to decimals and tenths, hundredths, etc. And this in turn will enable them perform basic calculations using numbers with decimals. Once they have learned how to calculate with fractions and convert them to decimals, all of this becomes unified into a much more complex schema that enables them to progress on to algebra, statistics and other more advanced topics.

However, for this to work, we need complete mastery at each of the preceding stages--right back to learning what numbers are. It is now increasingly accepted that in the early stages of learning about any subject--and in Maths, this will almost certainly include most if not all learning in primary school--teachers would be well advised to heed the seminal paper by Kirschner, Sweller and Clark: "Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching." [21] For present purposes, we will refer to it as "KSC".

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20 Eriksson, K., Helenius, O., & Ryve, A. (2018). Using TIMSS items to evaluate the effectiveness of different instructional practices. *Instructional Science*, 1-18.

21 Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational psychologist*, *41*(2), 75-86.

In short, the problem is that our working memories--that is, what we can hold in mind at any given moment-- have very limited capacity. Research suggests that we can only hold between four and seven facts in mind at once, and if not constantly re-visited, a fact will drop out of our working memory in a matter of seconds. Fortunately, if we know a topic really well and can bring it to mind instantly and automatically, it takes up no more room in our working memory than a single fact. This enables us to focus on new learning and integrate it into our existing knowledge. In other words, we build expertise incrementally, and the less we try to absorb at one time, the easier it is for us to learn.

Providing that this new knowledge is practised to the point where it can be recalled instantly and automatically, this in turn allows us to build increasingly complex schema of interconnected knowledge. As we gradually become experts, new knowledge is likely to make more connections with what we already know, and we need less guidance from an expert teacher. It is well to bear in mind that even undergraduates normally need a fair amount of expert guidance: that's why universities have lecture halls.

This has profound implications for the curriculum and for pedagogy. KSC is one of the most influential educational documents of the present century. As of 27 December 2018, Google Scholar reports that it has been cited 5,869 times in academic works. As of 9 January 2019, it has been cited 5,891 times--over the holidays, it was cited 22 times. Rosenshine has distilled the cognitive revolution into these principles for teachers--they apply to all academic subjects, not just Maths:

> 1. Begin a lesson with a short review of previous learning: Daily review can strengthen previous learning and can lead to fluent recall.
>
> 2. Present new material in small steps with student practice after each step: Only present small amounts of new material at any time, and then assist students as they practice this material.
>
> 3. Ask a large number of questions and check the responses of all students: Questions help students practice new information and connect new material to their prior learning.
>
> 4. Provide models: Providing students with models and worked examples can help them learn to solve problems faster.
>
> 5. Guide student practice: Successful teachers spend more time guiding students' practice of new material.
>
> 6. Check for student understanding: Checking for student understanding at each point can help students learn the material with fewer errors.
>
> 7. Obtain a high success rate: It is important for students to achieve a high success rate during classroom instruction.
>
> 8. Provide scaffolds for difficult tasks: The teacher provides students with temporary supports and scaffolds to assist them when they learn difficult tasks.

> 9. Require and monitor independent practice: Students need extensive, successful, independent practice in order for skills and knowledge to become automatic.
>
> 10. Engage students in weekly and monthly review: Students need to be involved in extensive practice in order to develop well-connected and automatic knowledge. [22]

On this last item, Rosenshine shies at the fence--we now know that the most effective kind of review is recall, which is far more effective that re-studying the material. [23] The effort of bringing something to mind is the most powerful means of securing it in long-term memory. And the most efficient means of eliciting that recall is a simple written test--although the correct use of closed questions can also work well. Tests and closed questions (questions designed to elicit a specific response) have long been strenuously opposed by progressive educators. The great irony of the Maths revolution is that its opponents are undoubtedly sincere in their desire to make our children capable of independent, self-directed study of Mathematics. So do we--but a substantial body of research suggests that they will never get there unless they receive efficient, fully-guided instruction in the early stages of their education.

Even though it takes a lot of time to achieve automaticity with each new topic and to consolidate it with existing schemata, at least it is time well-spent compared to the time wasted teaching knowledge and skills that are forgotten and need to be re-taught each year--which is what inevitably happens with an over-loaded spiral curriculum. Since the latter was popularised by Jerome Bruner in the 1970s, we have made great strides in understanding how learning is best consolidated so that it is instantly available for more complex or advanced tasks. Now, there can be no excuse for this dated relic of progressive ideology.

In theory, there is nothing to prevent schools from writing a curricular progression giving procedural knowledge priority over conceptual knowledge and abandoning the spiral structure. Yet so long as the advice of the NCETM remains closely aligned to present structure of the National Curriculum, it will take a very brave Maths curriculum leader to depart from it.

5. Constructivism:

The National Council of Teacher of Mathematics (NCTM) is the American equivalent of our NCETM. Both organisations advocate what is often termed as a 'constructivist' approach to teaching Maths--in 2001, the former set out their principles, which include the following:

> Reform-minded teachers pose problems and encourage students to think deeply about possible solutions. They promote making connections to other ideas within mathematics and other disciplines. They ask students to furnish proof or explanations for their work. They use different representations of mathematical ideas to foster students' greater understanding. These teachers ask students to explain the mathematics.

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22 Rosenshine, B. (2012). Principles of Instruction: Research-Based Strategies That All Teachers Should Know. *American educator*, *36*(1), 12.

23 Roediger III, H. L., Putnam, A. L., & Smith, M. A. (2011). 1 Ten Benefits of Testing and Their Applications to Educational Practice. *Psychology of Learning and Motivation-Advances in Research and Theory*, *55*, 1.

> Their students are expected to solve problems, apply mathematics to real-world situations, and expand on what they already know. Sometimes they work with other students. Sometimes they work alone. Sometimes they use calculators. Sometimes they use only paper and pencil... there is great benefit to allowing students to construct their own algorithms for addition and subtraction. [24]

The NCETM now acknowledges the impact of recent findings such as those summarised by KSC and Rosenshine and now admit that teachers actually have to teach if children are expected to learn. They have appropriated the language of 'mastery', yet at the same time support the existing progression in the National Curriculum and its fixation with conceptual knowledge. For all that they now admit that pupils should have automatic recall of number bonds, they still insist that mastery entails teaching concepts first:

> Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The structure and connections within the mathematics are emphasised, so that pupils develop deep learning that can be sustained. [25]

As is so often the case with expert opinion, it is based upon a hopelessly optimistic estimation of what can be achieved by even the most talented teachers--or, for that matter, the most talented pupils. For instance, the National Curriculum insists that by the age of 16, all pupils--not just the brightest--should be able to

> ...reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language. [26]

One can easily imagine the hollow laughter of any Maths teacher who has ever taught any of the bottom sets in Year 10 or Year 11. Even for the top sets, it's a tall order.

Admittedly, the Maths Hubs have only been around since 2014, and no doubt there is a certain amount of tension within the NCETM between constructivists and those who accept the more recent findings of the cognitive sciences. At very least, we can be sure that the Maths Revolution has progressed to the point where scholars like KSC can no longer be dismissed, and in any event there can be no doubt that conventional algorithms are now being taught--and taught well--in at least some primary schools. Not so long ago, this was strongly opposed by constructivists like Kaami and Dominick. They argued that

> Children in the primary grades should be able to invent their own arithmetic without the instruction they are now receiving from textbooks and workbooks...Algorithms not only are not helpful in learning arithmetic, but also hinder children’s development of numerical reasoning....We have two reasons for saying that algorithms are harmful: (1) They encourage children to give up their own thinking, and (2) they “unteach” place value, thereby preventing children from developing number sense....The persisting difficulty [with basic algorithms] lay in the column-by-column, single-digit approach that prevents children from thinking about multidigit numbers. [27]

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24 Constructivist Mathematics and Unicorns - National Council of Teachers of Mathematics

25 https://www.ncetm.org.uk/files/37086535/The+Essence+of+Maths+Teaching+for+Mastery+june+2016.pdf

26 https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/331882/KS4_maths_PoS_FINAL_170714.pdf

27 Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. *The teaching and learning of algorithms in school mathematics*, *19*, 130-140.

In fact, the columnar algorithms for addition, subtraction and multiplication make place value transparent, and it would take a very perverse teacher to "unteach" it. Ball et al conclude:

> Students should be able to use the basic algorithms of whole number arithmetic fluently, and they should understand how and why the algorithms work. Fluent use and understanding ought to be developed concurrently. These basic algorithms were a major intellectual accomplishment. Because they embody the structure of the base-ten number system, studying them can reinforce students' understanding of the place value system. [28]

However, the main problem with Kaami and Dominick's argument is that there is very little evidence that children's 'own thinking' is really their own, or even that it helps the 'development of numerical reasoning'. Norton administered tests similar to ours to pupils in Years 4, 5, 6 & 7 in Queensland, and found that

> ...those students who had mastered the traditional algorithms and recorded their working using the traditional algorithms were more successful than those who used alternative methods and recorded their working in ways that reflected these methods, especially when numbers become larger.

After analysing the workings of pupils who used alternative calculating strategies, Norton concluded that

> It was also evident that many of the students did not develop the intended deeper understanding of number and progression to effective algorithms was not attained. In summary, there was evidence that alternative methods were not developed intuitively, but were probably taught, and not in a coherent and systematic way that helped develop efficient algorithms; rather, the alternative methods tended to become a grab bag of procedures that lacked a deep structural underpinning. In this way the added variety of methods was achieving little more than giving students a further range of algorithms to mix up and make mistakes with. [29]

As we will see, our analysis of the workings of the pupils tested at Pendle Vale has led to the same analysis, and this is what is really at the heart of the present debate about teaching primary Maths. The NCETM is trying to have it both ways, even though the progression they advocate is in clear conflict with KSC.

6. Primitive strategies don't work:

Although there can be no doubt that Kamii and Dominick's hostility to basic algorithms is widely shared by maths educators, no one seriously pretends that pupils shouldn't be taught some means of solving mathematical problems. As Norton suggests, pupils are taught a range of primitive calculating strategies--all of which are less efficient than basic algorithms. Constructivists argue that by giving young children different strategies for addition, subtraction, multiplication, etc., they will learn to 'think mathematically' in order to determine

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28 Ball, D. L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J., Schmid, W., & Schaar, R. (2005). Reaching for common ground in K-12 mathematics education. *Notices of the AMS*, *52*(9), 1055-1058.

29 Norton, S. (2012). The use of alternative algorithms in whole number computation. *International Journal of Mathematics Teaching and Learning*, *26*, 2-16.

which strategy is most appropriate for solving a given problem. This not only conflicts with KSC but also with our analysis of pupils' errors on our tests at Pendle Vale.

1. Counting--this is most primitive calculating strategy of all. Being that this is an oral (or at least sub-vocal) activity, we weren't able to determine how many of the answers on our tests were arrived at by counting. However, the National Curriculum for primary Maths specifically uses the word 'count' or 'counting' no less than 18 times, and this doesn't take in into consideration the fact that addition and subtraction are usually taught with the use of a 'number line'. When subtracting 8 from 15, the pupil is taught to find 15 on a number line, and then count backwards 8 places.

In Year 1, pupils practise counting in multiples of 2, 5 and 10; in Year 2, it's multiples of 2, 3 and 5 (both backward and forward); by the end of year 3, they will also be able to count by intervals of 4, 50 and 100. Year 4 is a bonanza year--they learn to count in multiples of 6, 7, 8, 25, 1000 and even hundredths.

With what we now know about cognitive load, this practice is indefensible--although of course children do need to learn how to count accurately and automatically. Counting is not only a legendarily soporific activity, but when you are counting, your working memory is fully engaged and hence unable to reflect on any mathematical ideas save for getting the numbers in the correct order and articulating them. In other words, it acts like a mantra, blocking conscious thought.

In theory, counting activities instil 'number sense'--this will be discussed later. In fact they serve little purpose beyond delaying the acquisition of automatic recall of number bonds.
As we learned from an earlier 2016 research project with lower ability Year 9 pupils at Pendle Vale--this is described later-- only 3 out of the 24 pupils involved had anything like automatic recall of number bonds for addition at the start of our intervention; in other words, most of them had to count for any but the simplest sums, such as 2+2 or 1+5.

It would be difficult to think of any other activity more likely to kill a pupil's enthusiasm for Maths--or one which inhibits the development of the automaticity needed to progress to more advanced levels of understanding. As Wong and Evans conclude,

> Without procedural fluency and the ability to recall facts from memory, the student's focus during problem solving will be on basic skills rather than the task at hand, thus drawing attention away from the learning objectives of the task (Mercer & Miller, 1992). If the student cannot perform these basic calculations without the need to use calculators or other aids, higher-order processing in problem solving will be impeded. [30]

2. Primitive multiplication strategies: Although some pupils with low recall scores had good scores on the arithmetic test and others with good recall scores did poorly on arithmetic, overall ability and quality of instruction are almost certainly major factors. In general, there can be no question: the great majority of pupils with low arithmetic scores also had poor recall scores. In the 2018 cohort, of the 60 pupils with arithmetic scores <4, only 3 had recall scores >20.

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30 Wong, M, & Evans, D (2007) Improving basic multiplication fact recall for primary school students *Mathematics Education Research Journal*, *19*(1), 89-106. https://files.eric.ed.gov/fulltext/EJ776257.pdf

Perhaps even more significant are the primitive strategies pupils use for multiplying multi-digit numbers. These include repeated addition, doubling, and partitioning. One of the most frequently used was the grid method, which is a formalised version of partitioning. Our analysis demonstrated that these 'strategies' are not as effective as the basic algorithm:

Combined 2016 & 2017 intake:

Method:	Basic	Grid	Other
No. of pupils	167	98	76
% of pupils	48.9%	28.7%	22.3%
No. of correct answers for Q 3	101	37	12
% of correct answers for Q 3	60.5%	37.7%	15.8%
Mean score on test	6.95	4.88	4.22
% of pupils scoring <4	8.4%	25.5%	44.7%

2018 intake:

Method:	Basic	Grid	Other
No. of pupils	114	46	46
% of pupils	55.3%	22.3%	22.3%
No. of correct answers for Q 3	73	16	3
% of correct answers for Q 3	64.0%	34.7%	6.5%
Mean score on test	6.39	5.39	3.02
% of pupils scoring <4	13.1%	28.3%	63.0%

Of course, it may be possible that some primary schools only teach the basic algorithm *after* pupils have mastered more primitive methods, which would indicate that these were the most able pupils. As we will see from the analysis of responses on the percentage questions, this is extremely unlikely. However, the fact that so many schools are beginning to emphasise basic algorithms is encouraging.

3. Partitioning: In theory, partitioning doesn't sound too difficult. To solve our first percentage question--36% of 14-- the pupil should calculate as follows:

1. 10% of 14 = 1.4
2. 30% of 14 = 1.4 + 1.4 + 1.4 = 4.2
3. 1% of 14 = .14
4. 6% of 14 = .14 + .14 + .14 + .14 + .14 + .14 = .84
5. 4.2 + .84 = 5.04

However, a pupil from the top set in the 2018 cohort--one with fluent recall of number bonds--became hopelessly muddled with both percentage questions, as did the great majority of our pupils. The workings on the above question went as follows:

1. 10% of 14=1.4
2. 1.4 + 1.4 + 1.4 = 4.2 (awarded point by teacher)
3. Thinks that 1% will be 10% of 4.2 =.42
4. Adds .42 + .42 + .42 = 1.26
5. Doubles this-- 1.26 + 1.26 = 2.52
6. Forgets all about the 4.2 calculated in steps 1&2, and circles 2.52 as the answer. By sheer coincidence, this is exactly half of the correct answer.

With Q.11--47% of 563--this pupil actually multiplied a decimal number accurately, but then became hopelessly confused :

1. Same as steps 1&2 above, except this time the pupil multiplies 56.3 x 4 = 225.2 (awarded point by teacher)
2. Takes 10% of this--which is 22.52--and for some reason divides it by 6. The answer is 3.75 1/3, which is fine so long as you don't mind mixed whole numbers, decimals and fractions.
3. Multiplies the 3.75 x 10, but forgets to multiply the 1/3. Voila, the answer is 37.5 1/3! (off by a mere factor of 7).

This pupil clearly understood partitioning, decimals, division and multiplication--but even then, the partitioning method over-taxes the working memory of even the brightest pupils. Poor presentation also becomes a major factor when using complex, multi-step methods; pupils' workings almost invariably consists of random jottings. At least in this example, it was possible to determine the order of operations attempted, but even then it was not set out in any systematic way.

Of the 21 pupils in the 2018 cohort who answered Question 11 correctly, all but 9 were in the top set. For all that maths educators stress the importance of pupils being able to solve 'real world problems', one could almost be forgiven for thinking that the partitioning method was sponsored by Britain's notorious 'payday lenders'--who apparently charge interest rates of up to 2,000%.

7. Number sense:

All of the above strategies reflect a desire to impart 'number sense'. Few terms more accurately illustrate the gulf between mathematicians and maths educators. It is not one that will often be used by the former, nor has there ever been an agreed definition for it. The one offered by Case is probably as good a summary as we get:

> Number sense is difficult to define but easy to recognize. Students with good number sense can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical operations. They can represent the same number in multiple ways depending on the context and the purpose of this representation. They can recognize benchmark numbers and number patterns: especially ones that derive from the deep structure of the number system. ***They have a good sense of numerical magnitude*** (emphasis supplied). Finally, they can think or talk in a sensible way about the general properties of a numerical problem or expression—without doing any precise computation.[31]

Closer to home, the NCETM website explains number sense:

- research into the development of number sense is concerned with either the acquisition of number sense by young children or with the number sense that people require to function well in the world;

- hard to define ‘number sense’; the human competences and abilities indicating the existence of number sense that have been revealed by research;

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31 Case, R (April 1998) "A psychological model of number sense and its development", a paper presented at the American Educational Research Association, quoted in Gersten, R and Chard, D (2001) "Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities", www.ldonline.org

- pupils' number sense can become impaired when they are introduced to calculation-algorithms...ways of addressing that;
- ability to count carefully and accurately in different situations;
- providing opportunities for pupils to count in different ways, forwards and backwards... counting circles ...;
- counting errors ... eg counting the moves from one object to another instead of counting the objects;
- using fingers to support counting ... 'hands-on-the-table' policy to see which pupils are using fingers ... 'using fingers' as a strategy to move on from ... using fingers to develop an early sense of composition of numbers... [32]

Berch lists 30 different components of number sense that he found by trawling the literature, [33] but central to all of them is the ability to make reasonably accurate estimates of quantity and to spot unreasonable errors.

If our tests are anything to go by, primary maths teaching in England has proved a dismal failure. Using the criteria that an 'unreasonable error' is more than double (or less than half of) the actual answer, we looked at a sample of our results for our 2017 cohort. Of the 23 papers in Set 3, 22 contained at least one unreasonable error. Eleven papers contained 2 or more unreasonable answers.

It might be argued that being off by a factor of two is not 'unreasonable', but 18 answers were off by a factor of five or more, and of these 8 were off by a factor greater than ten. One pupil calculated 438 x 63 = 876,814, and another reckoned 47% of 563 at 11.26. The only pupil who avoided making any unreasonable errors did so by the simple expedient of not answering 7 of the 12 questions.

Predictably, Set 1 did much better, mostly by virtue of not making anywhere near so many errors. Even then, only 12 papers were free of unreasonable errors. In the other 15 papers, nine of the errors were off by a factor >10, and three of these were off by a factor >100.

The papers for Set 8 defy analysis. Even when pupils attempted a question, presentation was so poor that it was difficult to determine which number was intended as the answer. In the 8 papers, there were only 6 correct answers in total, which--by any criteria--demonstrates a total lack of number sense, whatever definition one chooses.

Ironically, one of the main arguments for abandoning traditional arithmetic teaching was that it was so tedious that it put children off Maths. Yet the following report from a secondary teacher frustrated by the glacial pace of their child's progress in Year 2 suggests that teaching 'number sense' is positively catatonic by comparison. After practising the Mathematical Reasoning paper for KS1 SATs, they looked at a homework assignment:

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32 https://www.ncetm.org.uk/public/files/96731840/2018_11_06_mathscpdchat+_summary.pdf

33 Berch DB. Making sense of number sense: implications for children with mathematical disabilities. J Learn Disabil. 2005;38:333–339 https://pdfs.semanticscholar.org/ec96/1459645f3473f0fc32e359c8d58462234ae7.pdf

> A lot of the actual questions were similar to the one [on the SATs paper] with two thermometers - one showing 35 degrees and the other 15 degrees and the question was to write down the difference in temperature. Bobby just hasn't been taught to see this as a simple subtraction of 35-15. He first started counting back from 35 to 15, then realised it would be easier to go the other way, counting up in 5s and raising a finger each time, until he had 4 fingers up, then counting up in 5s, 4 times, to get 20. My objection is that this is both pathetically easy, and also very cumbersome, so mistakes are made anyway. On more difficult ones, he wrote all the numbers down, so it took even longer and he made a number of simple mistakes, resulting in wrong answer.
>
> Kids who've done this even a few times should be taught to use column methods right away. Otherwise they are being restricted to needlessly simple problems. For example, Bobby can easily do sums like 58-29 using the column subtraction we've taught him, but will not be asked to do so for the rest of this school year, at least. He'll just get more of the same. He's being doubly held back because practising those harder sums would also help to cement his number bonds - and hence his mental arithmetic. Doing it 'their way' is just painful to watch. For bright kids (without teacher parents) this could be holding them back for years, but it's institutionalised by the NC and the mathematical reasoning tests they are working towards.

We don't know if Bobby's school is using the same approach as Pendle Vale's feeder schools, but the approach very closely tracks the suggested progression recommended by the NCETM. This is an egregious example of how children are held back by a curricular progression which prioritises conceptual development over basic declarative knowledge and procedural skills. As we have noted elsewhere, the 'safe' option for any school is to stick closely to this advice.

It must be stressed that neither the positive Ofsted reports of the feeder schools nor their KS2 Maths scores suggest that our pupils are in any way atypical. Also, there are two ways these scores can be interpreted in reference to 'number sense'. If, as most maths educators insist, number sense is a precondition for acquiring fluency, this would argue that the activities advocated by the NC and the NCETM don't instil it. If number sense is a *consequence* of acquiring fluency, it is equally obvious that only a tiny minority of our sample have acquired anything remotely resembling 'number sense'.

8. Catching up:

In secondary school, teaching the bottom sets can be difficult in any subject, but in highly structured subjects like Maths, the failure to learn the most basic knowledge is particularly problematic. Although there should never be an excuse for bad behaviour, it is difficult not to feel a little sympathy for pupils who've fallen way behind. Their days in school stretch out in an unending procession of boredom, futility and humiliation. It's one thing to provide 'enrichment' activities like school trips by way of compensation, but such palliative measures do little if anything to 'close the gap' academically.

As we have mentioned, the research culture at Pendle Vale is very strong--teaching staff are actively encouraged to read and participate in evidence-based research. As a senior science teacher, one of Colin McKenzie's first projects involved testing the basic maths skills of 18 lower-ability Year 9 students and devising a useful intervention to help them improve. The class was a notoriously difficult group, containing pupils with various 'special needs' and behavioural problems.

Many were just disengaged by science, and had been during their first three years in secondary education. Their targets were low (between Level 1 and 4 on a scale that goes up to 9), and most were failing to meet them. First, they were given a timed test to determine whether they could answer simple 2-digit sums without counting, and then they were given simple addition and subtraction problems involving two-and three-digit numbers. None of the pupils had maths skills adequate for even the simplest calculations needed for Chemistry and Physics.

For the next term, they participated in the *Fast Maths* project, with students practising and testing each other on knowledge of number bonds for addition and multiplication, as well as working through simple arithmetic problems including column addition and multiplication using standard algorithms. These sessions lasted 15 minutes and took place three times a week. A regular feature of these session were timed tests; pupils made rapid progress which proved a powerfully motivating factor.

The results were a revelation. The pupils not only made real, rapid and measurable progress in basic maths, but their whole attitude to learning was transformed. This previously 'difficult' group of students were arriving on time and getting straight to work. As their maths scores improved, so did their enthusiasm. They were learning something meaningful and they could see their own results getting better each week. Behaviour problems evaporated and their science results improved as their trust developed and 'engagement' became the default mode. Two years later – when this intervention had long finished--most of these pupils could still shoot maths answers straight back when challenged suddenly in corridors between lessons. And they did so with a smile – the learning had 'stuck'. They were proud to have been a part of such a successful innovation--one which had given them an unprecedented degree of self-respect.

The two most striking aspects of this experiment were that pupils were not put off by supposedly 'boring' drill, nor were they afraid of stopwatches or tests. If only the same were true of all teachers!

9. Recommendations:

All of the research cited in this paper is contested. This is the nature of research in education and indeed virtually any branch of academia you care to mention. Broadly speaking, research conducted by academics from education departments tend to be opposed to our views, and studies conducted under the aegis of psychology departments or other branches of the cognitive sciences tend to support us. Of course, there are many notable exceptions, not the least being the authors--who have post-graduate degrees in education.

Nonetheless, we believe that the only way to resolve contested issues is to create a level playing field, which will enable us to find out what approaches work best for all children--including the least advantaged and the least able. Undoubtedly, many educators will object to using objective tests as the principal measure of effectiveness--but the only alternatives are to use subjective measures or to simply give up. Not only is there a substantial body of research outlining the positive effects of tests, [34] but Andreas Schleicher, the director for education and skills at the Organisation for Economic Cooperation and Development (OECD) admits that "there's no correlation across countries between the prevalence of tests in a system and students' anxiety". [35]

34 Roediger et al, op cit
35 https://www.tes.com/news/no-evidence-more-testing-drives-pupil-anxiety-says-pisa-boss

The following suggestions could be implemented within the scope of existing law, and the financial cost would be minimal. They will not produce instant results--rather, they would set in train a virtuous cycle of improvement. Most crucially, they could transform the lives of disadvantaged children, many of whom now spend their first seven years in school without learning even the most basic arithmetic.

- Ofsted should stop criticising failing schools--which seldom results in lasting improvements and adds substantially to teachers' workload [36]--and start celebrating the schools that are most successful in closing the achievement gap in KS2 SATs and GCSEs. These schools should outline their progression and the pedagogies employed, and Ofsted should summarise the most successful practices and publish them.

- Pupils scoring <30 responses per minute on the Times Table Check should repeat the test at least once annually until they achieve that standard.

- The Standards and Testing agency should break down KS2 Arithmetic SATs results to reveal which questions were answered correctly by each pupil and this information should be available to secondary schools for the purposes of formative assessment and as a baseline measure for voluntary catch-up programmes. These results should also be available to parents.

- For the evaluation of progress on catch-up programmes, the Arithmetic paper on each year's KS2 Maths SATs should be made available to secondary schools that would like to provide evidence of the effectiveness of their programme.

- Funding should be provided so that teachers from the schools that have proved most successful in closing the gap can be paid to disseminate their strategies. This could take any number of forms, involving the Maths Hubs, ResearchEd, INSET, or publishing information on the DfE website. Contact with other schools should be a two-way street--even the best schools will benefit from feedback.

Admittedly, these proposals will take time to produce results, and ministers tend to prefer quick fixes that will have an impact before the next election. Yet at the same time, it would be difficult for progressive educators to thwart a measure which rewards teachers for doing well--instead of punishing those who fail.

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36 http://parliamentstreet.org/wp-content/uploads/2017/02/Free-Schools-For-A-Free-Society.pdf